WATER RESOURCES MANAGEMENT

Reservoir Operation

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The Problem Setting

- Reservoir operation is keyed to meeting goals with quantities released as well as meeting goals that involve the storage in the reservoir.

- The mathematical model of reservoir storage has rather universal appeal since so many human-made systems involve storage, inputs, and releases.

- That is what we have to do is to decide on the input side (how much to get in) or at the output side (how much to release).
General Similar Examples

- Blood banking involves acquisition, storage, and distribution

- The traffic intersection involves vehicle arrivals, storage in the street, and vehicle departures through the intersection

- Military parts inventory systems acquire parts, store them, and distribute them at times and places of demand

- In fact, the water reservoir can serve as a model for virtually any system that involves inventory
A Reservoir
The Problem Setting

The reservoir system that will be considered is shown below

\[ S_t = \text{Reservoir storage} \]
\[ Q = \text{Min hydropower requirement} \]
\[ E \text{ and } F = \text{Max and Min stream flow} \]
\[ \text{Release during month } t \]
\[ Y_t = \text{irrigation release} \]
\[ X_t = \text{stream release} \]
The Problem Setting

The reservoir has multiple functions such as:

1. **Supply irrigation water** to an area of large farms where a target irrigation flow is furnished for each month. This is the allocation of water that results in the best crop yields for the mix of crops being grown.

2. To **maintain the flow in the stream** below the reservoir within desirable bounds.

3. The **production of a minimum amount of hydroelectric energy** through a specified amount of water that is directed through the turbines.

4. **Minimizing the variation** in storage.
Policy Objectives for Reservoir Operation

Policy objectives could be the maximization of expected annual net benefits from downstream releases, reservoir storage volumes, hydroelectric energy and flood control.
The Basic Model

- Apparently, the basic relationship of mass balance in hydrology is linear.

- The mass balance equation for the reservoir says that the storage at the end of the current period is equal to the storage at the end of the previous period \((t - 1)\) less any releases during the current period plus inflows during the current period.

- That is,

\[
S_t = S_{t-1} - X_t - Y_t + I_t, \quad t = 1, 2, \ldots, 12
\]
The Basic Model

- The basic model also limits the storage or reservoir contents to the capacity of the reservoir:

\[ S_t \leq C_{\text{max}}, \quad t = 1, 2, \ldots, 12 \]

- Water in excess of capacity will be directed either to the irrigation area or released through the turbines to downstream reservoir stream flow

- In addition, there may be also a minimum or dead storage requirement

\[ S_t \geq S_{\text{min}}, \quad t = 1, 2, \ldots, 12 \]
The Basic Model

- It is important to ensure that water is not borrowed from initial storage

- To prevent borrowing water, a single constraint is appended to the model that forces the storage at the end of the 12th month of operation to equal or exceed the known initial storage

\[ S_{12} \geq S_o \]
Example Reservoir Operation Problem

- A farmer plans to develop water for irrigation. He is considering two possible sources of water: a gravity diversion from a possible reservoir and/or a pump from a lower river diversion (see the figure).

- Between the reservoir and pump site the river baseflow increases by 2 ac-ft/day due to groundwater drainage into the river.

- The river flow into the reservoir, the farmer’s water demand during each of two six-month seasons of the year, and costs are all given in the table.
Example Reservoir Operation Problem

- Revenue is estimated at $300 per year for each irrigated acre

- The annualized capital cost of the reservoir is estimated to be $10/ac-ft of capacity. Pumping costs will be $30/ac-ft of water pumped. All non-water related costs are estimated to be $50/acre

- Develop an LP model that could determine which sources should be developed and the optimal number of acres that should be planted
Example Reservoir Operation Problem

<table>
<thead>
<tr>
<th>Season, $t$</th>
<th>$Q_t$ (ac-ft)</th>
<th>Demand (ac-ft/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Example Reservoir Operation Problem

An LP model to solve this problem can be formulated as follows:

Max $Z = \text{Revenue} - \text{Costs}$

Subject to:

- Mass balance in the reservoir
- Flows $\leq$ capacity
- Flows $\geq$ demand
- storage $\leq$ reservoir capacity
Example Reservoir Operation Problem

Use the following notation:

- \( R_t = \) flow from the reservoir during season \( t \)
- \( P_t = \) flow from the pump during season \( t \)
- \( AC = \) acres planted
- \( S_{\text{max}} = \) maximum reservoir capacity
Example Reservoir Operation Problem

The objective function

- Min $Z = (300 - 50) \text{AC} - (10) S_{\text{max}} - (30) P_1 - (30) P_2$
Example Reservoir Operation Problem

Mass balance in the reservoir

\[ S_2 = S_1 + 600 - R_1 - SP_1 \]
\[ S_3 = S_2 + 200 - R_2 - SP_2 \]
\[ S_1 = S_3 \]
Example Reservoir Operation Problem

Flows less than the capacity

\[ P_1 \leq SP_1 + 2 \left( \frac{365}{2} \right) \]

\[ P_2 \leq SP_2 + 2 \left( \frac{365}{2} \right) \]
Example Reservoir Operation Problem

Water demand is fully covered

\[ R_1 + P_1 \geq 1.0 \ AC \]
\[ R_2 + P_2 \geq 2.0 \ AC \]
Example Reservoir Operation Problem

Reservoir storage should be less than the available capacity

\[ S_2 \leq S_{\text{max}} \]
\[ S_3 \leq S_{\text{max}} \]
Example Reservoir Operation Problem

Lingo Solution

! The objective function is to maximize the benefit

Max = (300 - 50) * AC - 10 * Smax - 30 * P1 - 30 * P2;

! Mass balance constraints;

S2 = S1 + 600 - R1 - SP1;
S3 = S2 + 200 - R2 - SP2;
S1 = S3;

! Pumped water;

P1 <= SP1 + 2 * (365/2);
P2 <= SP2 + 2 * (365/2);

! Demand should be fulfilled;

R1 + P1 >= AC;
R2 + P2 >= 2 * AC;

! Storage capacity;

S2 <= Smax;
S3 <= Smax;
Smax = 300;
S1 = 100;
Example Reservoir Operation Problem

Lingo Solution

Global optimal solution found.
Objective value: 81675.00
Total solver iterations: 5

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>382.5000</td>
<td>0.000000</td>
</tr>
<tr>
<td>SMAX</td>
<td>300.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>P1</td>
<td>0.000000</td>
<td>30.000000</td>
</tr>
<tr>
<td>P2</td>
<td>365.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>S2</td>
<td>300.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>S1</td>
<td>100.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>R1</td>
<td>382.5000</td>
<td>0.000000</td>
</tr>
<tr>
<td>SP1</td>
<td>17.5000</td>
<td>0.000000</td>
</tr>
<tr>
<td>S3</td>
<td>100.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>R2</td>
<td>400.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>SP2</td>
<td>0.000000</td>
<td>30.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Row</th>
<th>Slack or Surplus</th>
<th>Dual Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>81675.00</td>
<td>1.000000</td>
</tr>
<tr>
<td>2</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>3</td>
<td>0.000000</td>
<td>125.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.000000</td>
<td>125.0000</td>
</tr>
<tr>
<td>5</td>
<td>382.5000</td>
<td>0.000000</td>
</tr>
<tr>
<td>6</td>
<td>0.000000</td>
<td>95.000000</td>
</tr>
<tr>
<td>7</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>8</td>
<td>0.000000</td>
<td>-125.0000</td>
</tr>
<tr>
<td>9</td>
<td>0.000000</td>
<td>125.0000</td>
</tr>
<tr>
<td>10</td>
<td>200.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>11</td>
<td>0.000000</td>
<td>115.000000</td>
</tr>
<tr>
<td>12</td>
<td>0.000000</td>
<td>125.0000</td>
</tr>
</tbody>
</table>
Example (Past Exam)

- You have three reservoirs interconnected as shown in the figure.

- The reservoirs are located in agricultural area. As a water resources manager, you are requested to develop a release strategy for the three reservoirs (that is to determine both SP and R values for the three reservoirs for the two time steps) in order to maximize revenue from planting the three agricultural areas.

- Note that the area for agriculture is constant through out the two time steps. Data is summarized in the table.
### Example (Past Exam)

<table>
<thead>
<tr>
<th>Time step</th>
<th>Benefit ($/m^2$) for each area</th>
<th>Demand (m³/m²)</th>
<th>Inflow (m³)</th>
<th>Capacity</th>
<th>Initial Storage (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1  2  3</td>
<td>Area 1  Area 2  Area 3</td>
<td>RS 1  RS 1  RS 2  RS 3</td>
<td>RS 1  RS 2  RS 3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10  15  5</td>
<td>1  1  1</td>
<td>200  50  110  70</td>
<td>40  50  60</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>30  25  40</td>
<td>1  2  2</td>
<td>250  50  110  70</td>
<td>-  -  -</td>
<td></td>
</tr>
</tbody>
</table>
Example (Past Exam)
Example (Past Exam)

Maximize revenue:

\[
\text{Max} = \text{Benefit11} \times AC1 + \text{Benefit12} \times AC1 + \text{Benefit21} \times AC2 + \text{Benefit22} \times AC2 + \text{Benefit31} \times AC3 + \text{Benefit32} \times AC3.
\]

Benefit:

\[
\begin{align*}
\text{Benefit11} &= 10; \\
\text{Benefit12} &= 30; \\
\text{Benefit21} &= 15; \\
\text{Benefit22} &= 25; \\
\text{Benefit31} &= 5; \\
\text{Benefit32} &= 40;
\end{align*}
\]

Initial storages:

\[
\begin{align*}
S11 &= 40; \\
S21 &= 50; \\
S31 &= 60; \\
t &= 1;
\end{align*}
\]

Water demand:

\[
\begin{align*}
\text{DEMAND11} &= 1; \\
\text{DEMAND12} &= 1; \\
\text{DEMAND21} &= 1; \\
\text{DEMAND22} &= 2; \\
\text{DEMAND31} &= 1; \\
\text{DEMAND32} &= 2;
\end{align*}
\]

Inflow to reservoir #1:

\[
\begin{align*}
I11 &= 200; \\
I12 &= 250;
\end{align*}
\]

Reservoir capacities:

\[
\begin{align*}
S1\text{max} &= 50; \\
S2\text{max} &= 110; \\
S3\text{max} &= 70;
\end{align*}
\]

Demand requirement:

\[
\begin{align*}
R11 &= AC1 \times \text{DEMAND11}; \\
R12 &= AC1 \times \text{DEMAND12}; \\
R21 &= AC2 \times \text{DEMAND21}; \\
R22 &= AC2 \times \text{DEMAND22}; \\
R31 &= AC3 \times \text{DEMAND31}; \\
R32 &= AC3 \times \text{DEMAND32};
\end{align*}
\]

Reservoir capacity:

\[
\begin{align*}
S12 &= S1\text{max}; \\
S13 &= S1\text{max}; \\
S22 &= S2\text{max}; \\
S23 &= S2\text{max}; \\
S32 &= S3\text{max}; \\
S33 &= S3\text{max};
\end{align*}
\]

Continuity principle:

\[
\begin{align*}
SP11 &= I11 + I12; \\
SP12 &= I12 + I21 + I22; \\
SP21 &= I21 + I22 + I23; \\
SP22 &= I22; \\
SP31 &= I31 + I32 + I33; \\
SP32 &= I32; \\
SP33 &= I33;
\end{align*}
\]

No borrowing:

\[
\begin{align*}
S13 &= S11; \\
S23 &= S21; \\
S33 &= S31;
\end{align*}
\]